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THE CONSTRUCTION OF A PHASE - SENSITIVE AMPLIFIER FOR THE DETECTION OF NUCLEAR QUADRUPOLE RESONANCE

A THESIS

Presented to

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Edwin Hardy Davidson

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THE CONSTRUCTION OF A PHASE-SENSITIVE
AMPLIFIER FOR THE DETECTION OF NUCLEAR
QUADRUPOLE RESONANCE

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Date Approved by Chairman: May 31, 1954

PREFACE

While investigating the subject of nuclear quadrupole resonance in a Summer Quarter, 1952, Graduate Lab, it became apparent that the future searching for quadrupole resonance lines would be facilitated by the use of a phase-sensitive amplifier. The author would like to express his appreciation to Drs. T.L. Weatherly and J.Q. Williams both for suggesting the thesis topic and for supplying many hours of constructive consultation.

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NOTE: Resistance, inductance, and capacitance values shown on illustrations are expressed in OHMS, HENRIES, and MICROFARADS respectively. The symbols M, K, and u represent multiplying factors of 10^{-3}, 10^{3}, and 10^{-6}, respectively.

THESIS ABSTRACT

THE CONSTRUCTION OF A PHASE-SENSITIVE AMPLIFIER FOR THE DETECTION OF NUCLEAR QUADRUPOLE RESONANCE

(54 pages)

by

Edwin Hardy Davidson

Work done in Summer Quarter, 1952, in Graduate Laboratory on nuclear quadrupole resonance indicated the need of an amplifier which would be highly sensitive only to signals of a predetermined frequency and phase. It was hoped that the construction of such a phase-sensitive amplifier would greatly improve the resonance signals already observed on an oscilloscope and would make possible the detection of much weaker signals.

Using a simple regenerative oscillator similar to the type used by Ralph Livingston for the detection of nuclear quadrupole resonance, the sample was surrounded by Helmholtz coils which created magnetic field when half wave rectified current flowed through the coils. Since quadrupole resonance signals cannot be detected in a polycrystalline sample in the presence of a magnetic field, the Helmholtz coils act as a switch which turned the quadrupole resonance signal on and off at a fixed frequency. To detect this signal only and to diminish moise, a modification of a phase sensitive detector circuit described by N. A. Schuster was used.

A 60 cycle reference signal is fed into the grids of a 6SN7 twin-triode tube. The reference signal serves to operate the twin-triode as a switch with each triode section alternately conducting every half-cycle. Conduction through the triode switch tubes is controlled by a 6SH7 pentode into which the reference signal is fed and since the quadrupole resonance signal is turned on and off at the same frequency as the 6SN7 switching triodes, the triodes plus the pentode form a unit which is sensitive to the quadrupole resonance signal, but not to signals of different frequencies and phase than the reference signal. This means that the quadrupole resonance signal is amplified without appreciably amplifying random noise.

The overall phase-sensitive amplifier circuit contains two pentode preamplifiers and a cathode-degenerative type selective amplifier to boost the signal before reaching the phase sensitive detector. A filter network and a post-amplifier follow the phase sensitive detector unit and the output leads are connected to an Esterline-Angus recording milliameter.

Comparative photographs of the same quadrupole resonance line as observed on an oscilloscope without the use of the phase-sensitive amplifier and as observed on the Esterline-Angus recorder using the phase-sensitive amplifier show conclusively that the phase sensitive amplifier greatly improves the signal-to-noise ratio. The main recommendation made is that a frequency other than 60 cycles be tried as the reference signal since this might serve to further decrease the noise level.

CHAPTER I

INTRODUCTION

Our knowledge of the atomic nucleus is far from complete at the present time, however, a few properties of nuclei seem to be definitely established.

The nucleus is a very small positively charged body which contains most of the mass of the atom. The results of atomic spectra indicate that nuclei rotate about their center of mass with angular momentum P given by the equation

$$P = \sqrt{I(I+1)}\,\hbar \tag{1}$$

The quantity I, called the nuclear spin, is a constant for a particular nucleus and may take integral or half integral values.

Due to the finite extent of its positive charge a spinning nucleus constitutes a circulating current and as a result possesses a magnetic dipole moment μ. If the atom containing this nucleus is placed in a magnetic field in the z-direction of magnitude H, it will have energy

$$E = E_o - \mu H \cos \theta \tag{2}$$

where E_o is the energy in zero field and θ is the angle between μ and H. According to the quantum theory a nucleus with spin

I in the field H can assume only those orientations for which the z-component of angular momentum is given by

$$P_z = m_I h \tag{3}$$

where m_I may take the values

$$m_I = I,\ I-1,\ I-2,\ \ldots\ -I \tag{4}$$

Thus cos θ takes only the values

$$\cos\theta = \frac{m_I}{\sqrt{I(I + 1)}} \tag{5}$$

Each value of m_I corresponds to a different value of energy, therefore the original energy level E_o is split into $2I + 1$ separate energy levels by the interaction of the magnetic dipole moment u with the magnetic field H.

If a large number of these nuclei are placed in a magnetic field H and subjected to r-f electromagnetic radiation of the proper frequency transitions will occur between $2I + 1$ levels according to the selection rule,

$$m_I = \pm 1 \tag{6}$$

Since the lower levels are more densely populated at thermal equilibrium, there will be more transitions to higher energies than in the reverse direction with the result that energy is absorbed from the r-f electromagnetic field. The absorption frequency is given by

$$\nu = \frac{\Delta E(\mu)}{h} \tag{7}$$

where $\Delta E(\mu)$ is the energy difference between the two levels involved as computed from (2). By measuring this absorption frequency the nuclear magnetic dipole moment μ can be calculated. This absorption phenomenon is referred to as nuclear magnetic resonance.

Theoretical investigations of nuclear properties predict a nuclear electric dipole moment of zero, and this prediction is confirmed by experiment. Higher order electric moments are possible however. Nuclei with spin greater than $\frac{1}{2}$ are found to have electric quadrupole moments.

The electric quadrupole moment of a nucleus is an indication of the shape of the nuclear charge distribution. This charge distribution is symmetric about the spin axis and the quadrupole moment eQ is given by the expression

$$eQ = \int_i r_i^2 \,(3 \cos^2 \theta_{ii} - 1)\, de_i \tag{8}$$

where r_i is the distance from the center of charge to the charge element de_i and the integral is taken over the entire nucleus. This equation is derived in Appendix A. A positive quadrupole moment indicates a prolate charge distribution and a negative moment indicates an oblate charge distribution.

When a nucleus with a quadrupole moment eQ is placed in an inhomogenous electric field such as that produced by the electronic structure of the atom of which it is a part, then its energy will depend on its orientation. Due to quantization of the nuclear spin only certain orientations are allowed. This results in quantization of the quadrupole energy, splitting the nuclear energy level into a number of components. For a nucleus with electric quadrupole moment eQ in an electric field which is rotationally symmetric about the z-axis the allowed values of the quadrupole energy are given by

$$E\ (Q) = \frac{eQ\ V_{zz}}{4\ I(2I-1)} \left[3m_z^2 - I(I + 1)\right] \tag{9}$$

where $V_{zz} = \frac{\partial^2 V}{\partial z^2}$. This equation is derived in Appendix B. Since the quadrupole energy is a function of m_z^2, all levels are doubly degenerate except the one for $m_z = 0$. Thus there are $I + 1$ levels when I is integral and $I + \frac{1}{2}$ levels when I is half-integral.

By subjecting these nuclei to an r-f electromagnetic field of proper frequency transitions are produced according to the selection rule

$$\Delta\ m_i = \pm 1 \tag{10}$$

These transitions are induced by the interaction between the nuclear magnetic moment and the r-f magnetic field applied just as in the case of nuclear magnetic resonance. The r-f frequency for which transitions occur is determined by the relation

$$\nu = \frac{\Delta E(Q)}{h} \tag{11}$$

where $\Delta E(Q)$ is the energy difference between the two levels involved as computed from (9). When such transitions are induced energy is absorbed from the r-f field. By detecting this absorption and measuring the absorption frequency one can compute the quadrupole coupling constant eQV_{zz}. This absorption phenomenon is referred to as nuclear quadrupole resonance.

It is the purpose of this thesis to describe the construction and use of a phase sensitive amplifier for the detection of nuclear quadrupole resonance.

CHAPTER II

HISTORY OF QUADRUPOLE RESONANCE DETECTION

Until 1951, the only observable quadrupole effects had been those secondary ones imposed on the observations of some other type of spectra. H. S. Dehmelt and H. Krueger (1) were the first to demonstrate that the quadrupole moment-field gradient interactions could be observed directly. These direct transitions were observed for chlorine in solid trans-dichloroethylene, each chlorine isotope giving rise to a single absorption line. Similar spectra were subsequently observed for the iodine nucleus in several iodine compounds.

The method that Dehmelt and Krueger chose for observation of the quadrupole resonance lines was similar to the method proposed by Arthur Roberts (2) for the measurement of proton magnetic resonance lines; namely, the use of a superregenerative oscillator. In observing nuclear magnetic resonance by Robert's method, it is necessary to supply a large uniform magnetic field which interacts with the proton magnetic moment. There is no need, however, for such a field in quadrupole resonance detection since the quadrupole moment interacts with the electric field gradient furnished by the electronic charge distribution of the atom.

Ralph Livingston (3), soon after the publication of Dehmelt and Krueger's observation, developed a low frequency spectrometer patterned after proton magnetic resonance equipment described by N. J. Hopkins (4). In magnetic resonance spectrometers it had been almost universal practice to modulate sinusoidally the magnetic field in which the sample is placed. Livingston's modification of the somewhat standardized equipment was to replace the magnetic modulation by frequency modulation of the r.f. oscillator with a vibrating condenser placed in the tank circuit. He then attempted to observe proton resonance in a uniform magnetic field. After eliminating the resulting amplitude modulation which at first obscured observations, he was able to observe good proton resonance signals. After this initial development, it was possible to remove the magnetic field completely and observe pure quadrupole transitions.

Livingston's r.f. oscillator unit was a triode-connected simple regenerative oscillator with self detection. It was operated entirely from batteries to minimize hum and many of the components were war surplus items. A block diagram of Livingston's circuit is shown in Figure 1 along with a wiring diagram of his oscillator unit in Figure 2.

Although the simple regenerative oscillator used by Livingston and the superregenerative oscillator used by Dehmelt and Krueger were technically different, the methods of detection were similar since in both an absorption in the

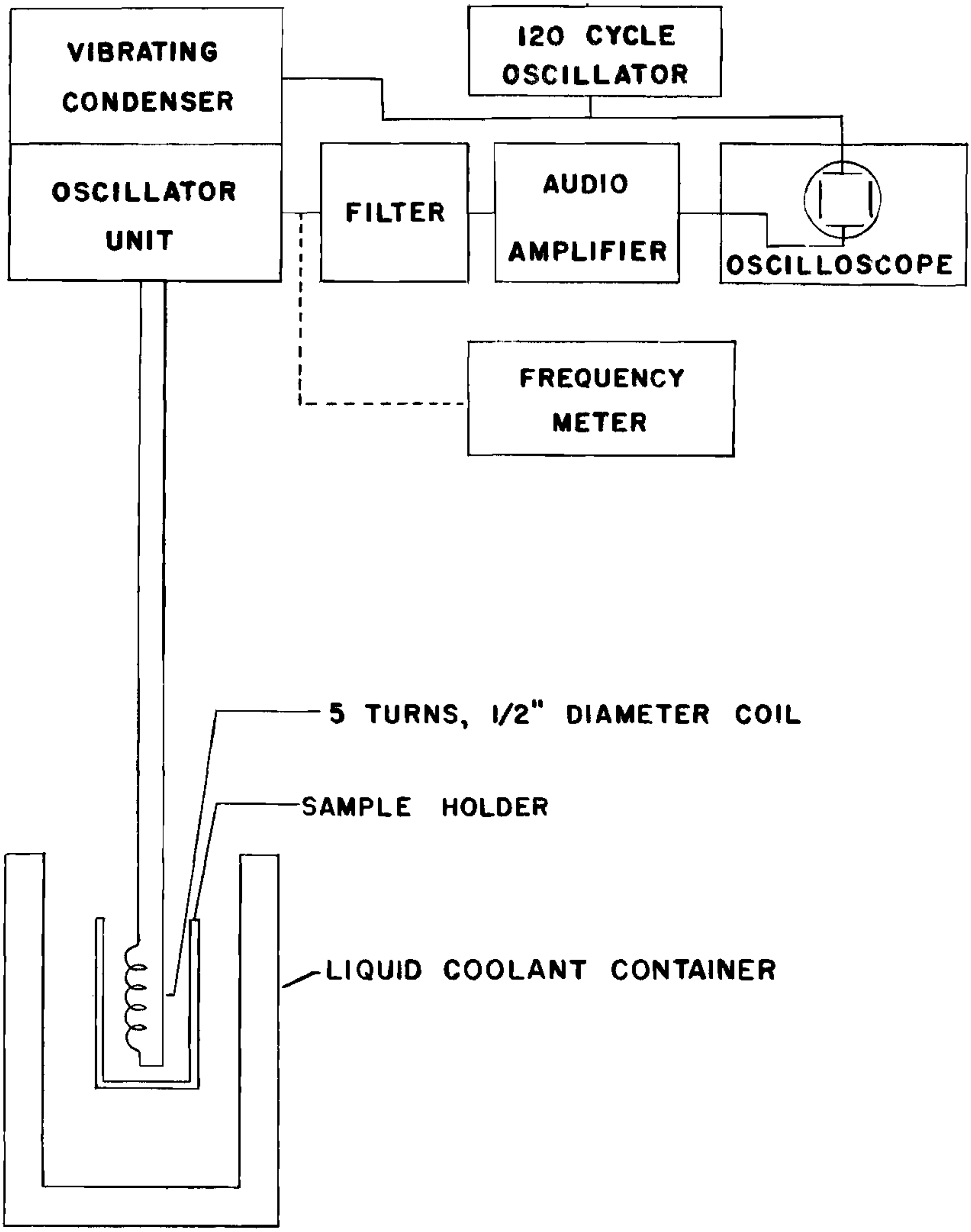


FIGURE 1 - BLOCK DIAGRAM OF LIVINGSTON'S DETECTOR

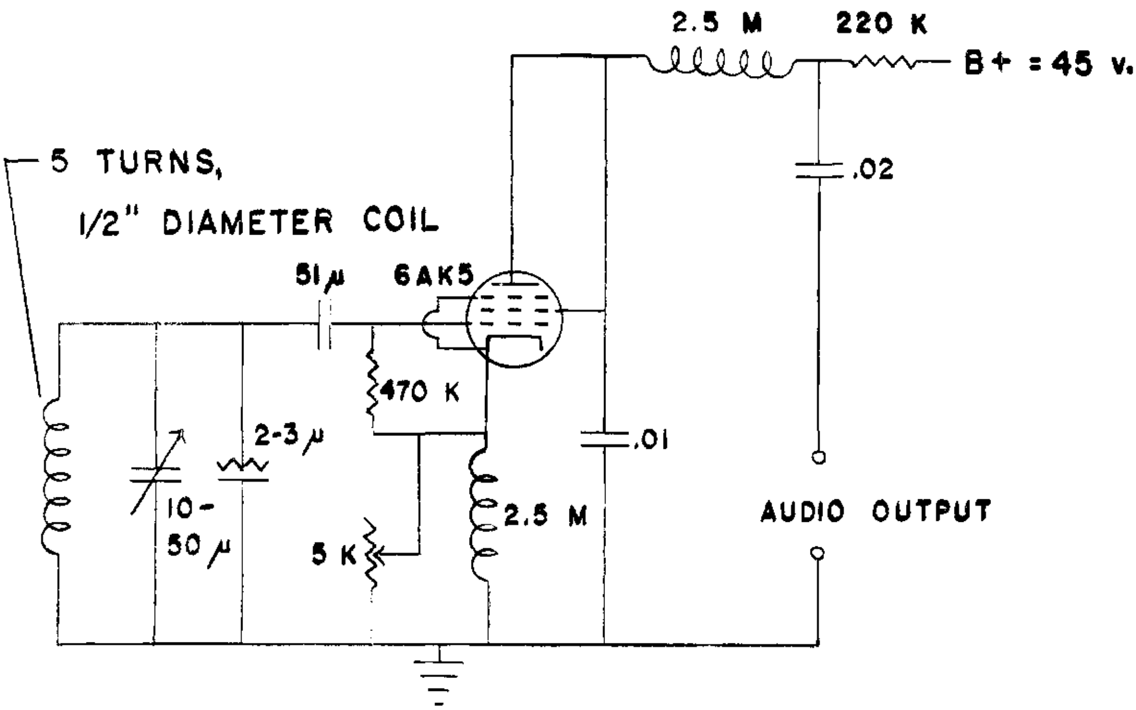


FIGURE 2 - LIVINGSTON'S FREQUENCY-MODULATED OSCILLATOR

grid circuit was amplified and observed on an oscilloscope. Both placed their sample in the vicinity of the tank coil which formed a part of the grid circuit. (Dehmelt and Krueger's sample was inserted in the coil while Livingston's coil was immersed in the sample.)

The oscillating current in the tank coil creates an alternating magnetic field and absorption occurs when the frequency is equal to E/h where E represents the energy difference between two energy levels, and h is Planck's constant. This magnetic field supplies the energy necessary to produce transitions from the lower energy level to the higher one, increasing the population in the higher level. If the coil continues to supply energy to the lower levels, it might seem that eventually all the nuclei in the lower energy orientation would be raised to a higher energy level so that, after a time, no more energy absorption from the coil could occur. However, there is a transfer of energy from the higher level to the crystal lattice structure, creating lattice vibrations which tend to neutralize the energy excess. Continuous radiation at the resonance frequency appears to create a sort of pseudo-equilibrium with energy taken from the magnetic field, added to the higher energy level, and dissipated through the crystal structure with the nuclei returning to the lower level.

This continuous withdrawal of energy from the magnetic field manifests itself as an increase in the real component of the impedance of the oscillator coil producing the field. The similarity of conditions to those arising in nuclear proton resonance detection led Dehmelt and Krueger and Livingston to use essentially the same methods of observation. In both systems the oscillator was frequency modulated by vibrating one plate of a condenser which formed part of the tank circuit. If the oscillator is tuned to the quadrupole resonance frequency and the condenser plate vibrated at 60 cycles per second, the oscillator frequency will then pass through the quadrupole resonance frequency twice each cycle. As it passes through the quadrupole resonance frequency energy is absorbed by the sample causing an increase in plate current which can be detected by observing the voltage drop across the plate load resistance.

By making connections to ground and to the negative side of the load resistance as shown in Figure 2, one obtains an output voltage proportional to the plate current. If the oscilloscope beam is swept horizontally with a 60 cycle saw-tooth voltage and if the oscillator output voltage is applied to the vertical deflection plates one obtains two pips on the face of the oscilloscope when the oscillator frequency is adjusted so that it sweeps through the quadrupole resonance. Each pip indicates an absorption of energy by the sample as the oscillator frequency passes through the quadrupole resonance, one occuring when the frequency is increasing and the other occuring when the frequency is decreasing.

Livingston (5) observed that piezoelectric substances under certain conditions would give rise to absorption spectra which in some cases might be confused with nuclear quadrupole lines. The problem of distinguishing one line from the other turned out to have a simple solution. By applying a constant magnetic field to the sample the nuclear energy levels are split into several components as explained earlier. The magnitude of this splitting is determined by the strength of the magnetic field and the orientation of the crystal. Thus the application of a magnetic field to a single crystal splits the quadrupole resonance line into several weaker components. In the case of a polycrystaline substance the line splitting for each crystal is different, resulting in a broadening of the observed line. This broadening of the quadrupole resonance line is accompanied by a decrease in intensity. Livingston, in a typical experiment with $CHCl_3$, found that a field of 12 gauss was sufficient to erase the lines. Piezoelectric lines, not sensitive to magnetic fields, were not affected.

The apparatus of Livingston, and Dehmelt and Krueger, contained provision for cooling the sample being investigated. By cooling the sample to liquid nitrogen temperature, one increases the number of nuclei in the lower energy level which increases the absorption intensity. Dehmelt and Krueger (6) demonstrated that the frequencies of the observed lines increase with decreasing temperature due to the effect of lattice vibrations on the electric field gradient. It is important in the resonance observation that there be no gradient in temperature of the heat bath surrounding the sample. In most line searching the sample was immersed in a beaker of liquid air which provided a uniform temperature of about 77°K. It was found in the chloromethanes that cooling from 77°K to 20°K resulted in a frequency increase of roughly 0.5% as well as an increase in the signal-to-noise ratio. At this frequency lattice vibrations were so thoroughly frozen that cooling from 20°K to 4°K brought about very little additional increase in frequency, and this change was accompanied by a loss in signal-to-noise ratio.

CHAPTER III

THE PHASE - SENSITIVE DETECTOR

A distinct disadvantage in the use of the Livingston or Dehmelt and Krueger apparatus for the observation of weak spectral lines is the fact that a wide band pass amplifier must be used between the detector and the oscilloscope. Such an amplifier is necessary since in a rapid search for new spectral lines response must be given to a wide range of frequencies. Such an amplifier, being sensitive to a wide band of signal frequencies, is also sensitive to a wide band of noise frequencies, which serves to decrease the signal-to-noise ratio.

The purpose of this thesis is to describe a method of detecting the signals using a phase sensitive detector, which is sensitive only to signals of a predetermined frequency and phase. There are several different circuits which accomplish this. The one used here is similar to the circuit designed by N. A. Schuster (7) to be used in the detection of nuclear magnetic resonances. Schuster's circuit is shown in Figure 3.

The input signal is fed into only one current determining tube while the reference signal in Figure 3 is injected into two triodes, the purpose being to switch the current alternately from one load resistance to another. However, the

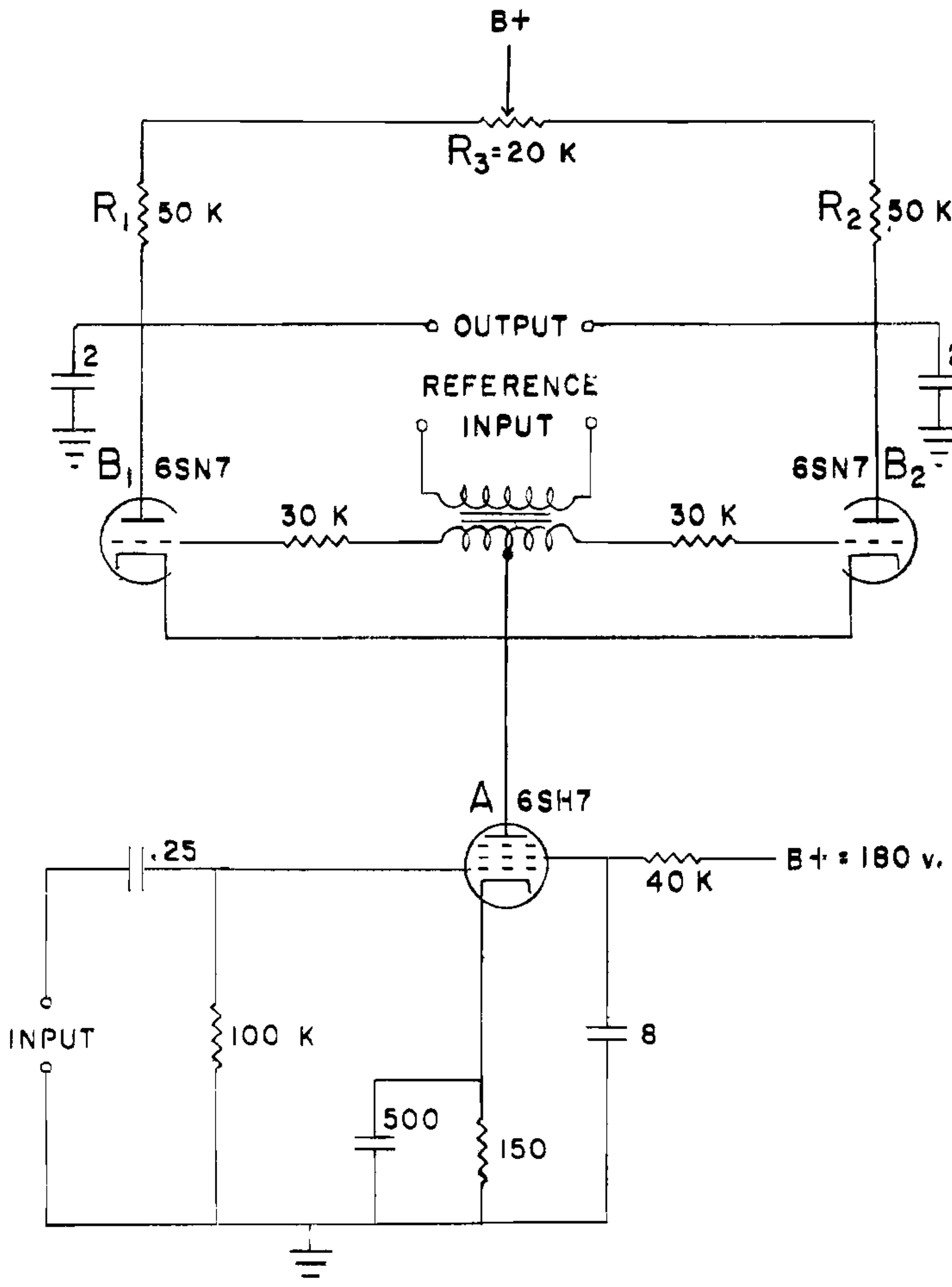


FIGURE 3 - SCHUSTER'S PHASE-SENSITIVE DETECTOR

variations in these switch tubes have been reduced to second order. That is, if a one percent change should occur in the plate resistance of one triode section, the resulting plate current change is less than .01 percent since the plate resistance of the pentode is more than 100 times greater than the resistance of the switch tube. The only elements which can have a first order effect on the balance are R_1, R_2 and R_3.

Schuster's circuit, with modifications indicated in Figure 4 appears as a part of the final constructed circuit for quadrupole resonance detection. It is the heart of the apparatus and consists of the 6SH7 pentode amplifier tube and the 6SN7 twin triode tube along with their associated circuit components. The twin triodes switch the plate current of the pentode from one load resistance to another. The reference signal applied to the grids of the 6SN7 is sufficient to cut off the plate current during the negative half-cycle and during the positive half-cycle the grids draw grid current allowing the grid voltage to be only slightly positive. The pentode is then connected to a load resistance through that half of the switch which happens to be conducting for that particular half-cycle and to the other load resistance during the other half-cycle.

The operation of the system is easily explained using the voltage waveforms given in Figure 5(a). First, no signal is applied to the grid of the 6SH7(e_g = 0). However, there

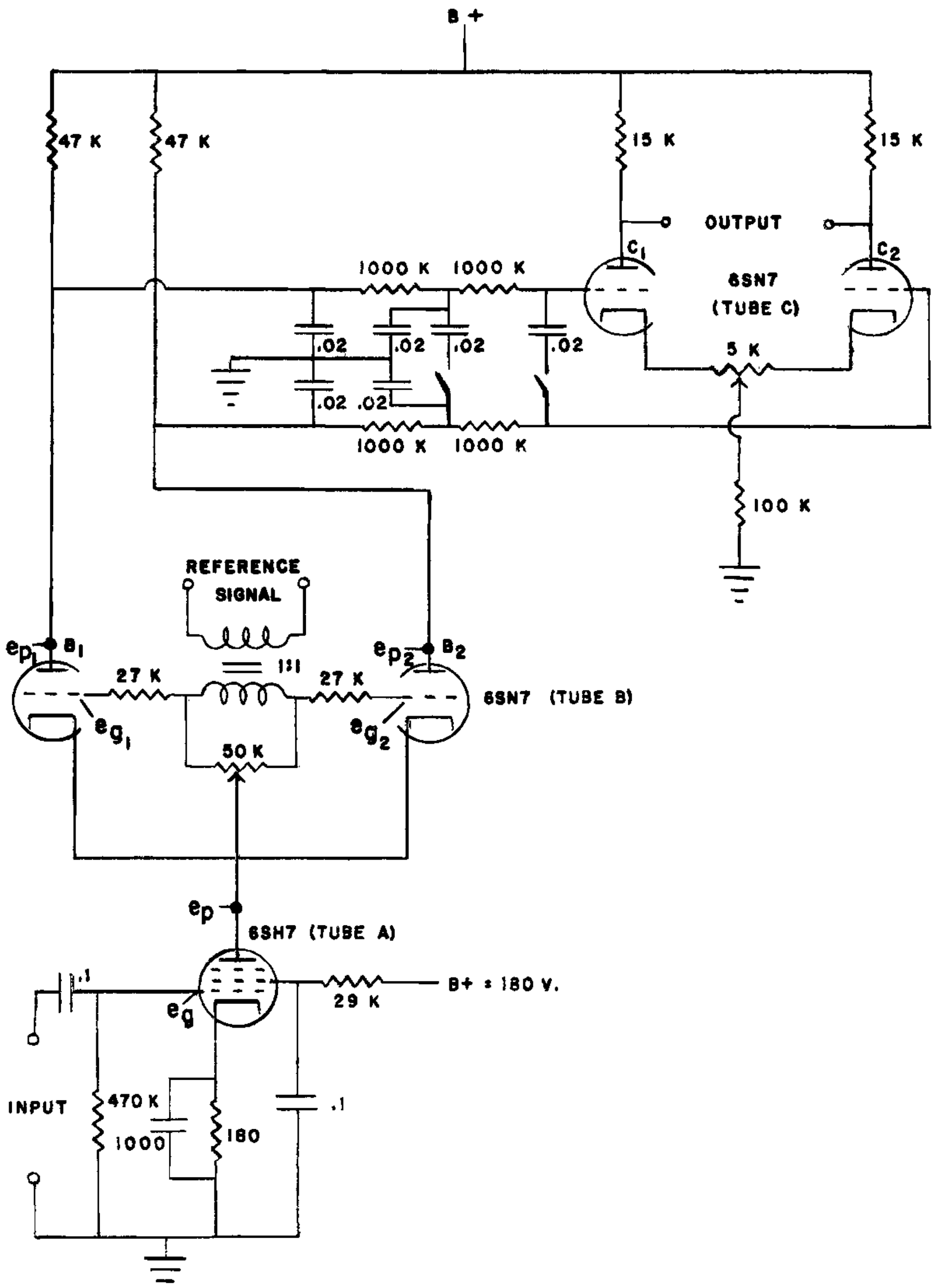


FIGURE 4 - MODIFIED SCHUSTER CIRCUIT

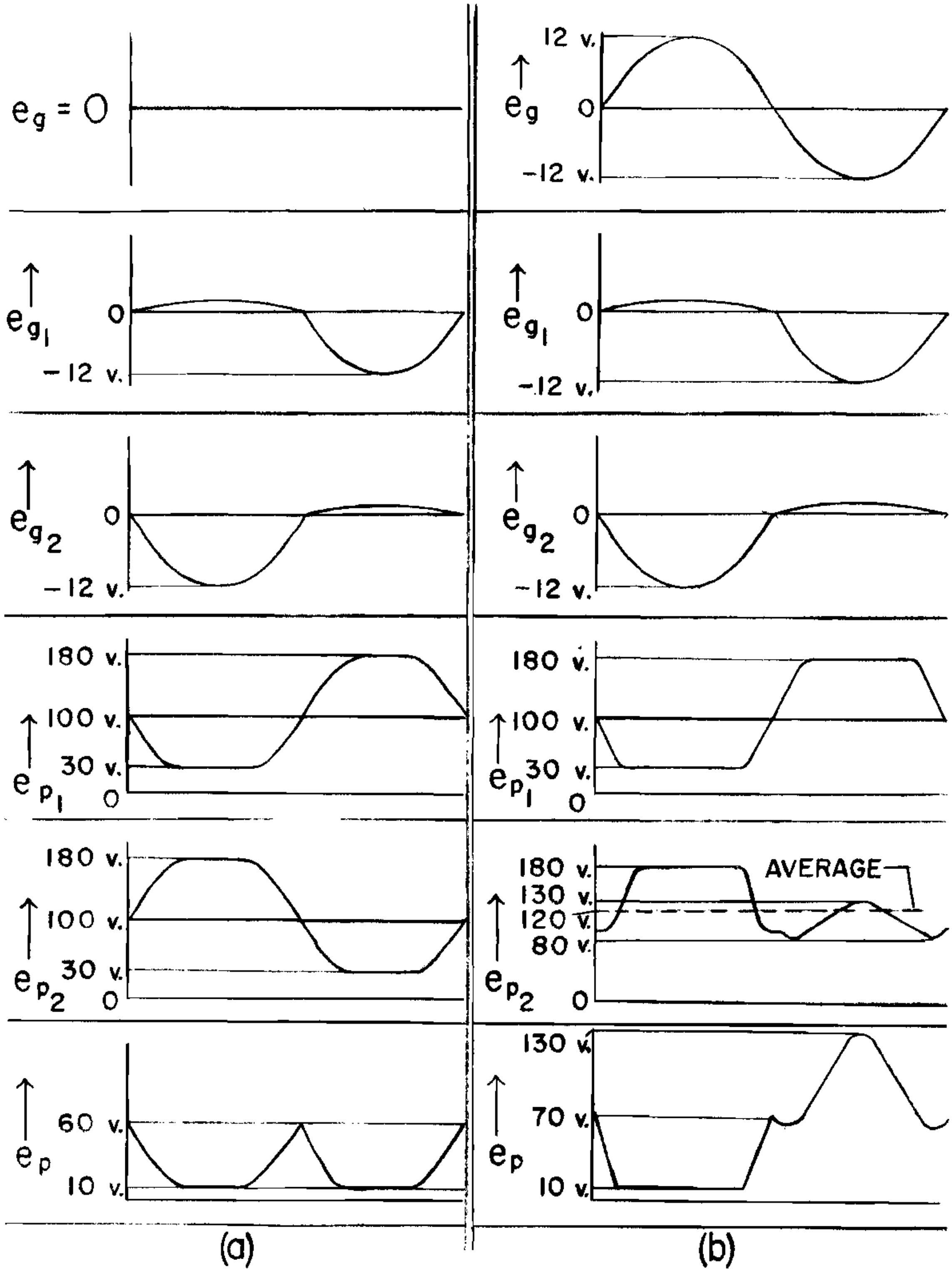


FIGURE 5-GRID AND PLATE VOLTAGE WAVEFORMS

is a reference voltage of 24 volts peak to peak applied to the switch tubes. The grids of the switch tubes draw current on the positive half-cycle and the positive value of the grid voltage is held near zero as is shown in the waveforms for e_{g1} and e_{g2} . The current in each switch tube is cut off for a portion of the negative half-cycle as is shown by the fact that the plate voltage curves, e_{p1} and e_{p2} , are flattened at 180 volts which is the B supply voltage. The resulting plate voltages of the switch tubes are approximate square wave voltages, one being 180^{o} out of phase with the other. Assuming that the 6SN7 used for the switch tube has been selected so that the two halves have the same characteristics, then by adjusting the size of the reference voltage applied to each grid the average values of the plate voltages can be made to be equal. A d.c. meter connected from one plate to the other of the switch tube would indicate no difference in potential.

The plate voltage of the 6SH7 is given by e_p . The half of the switch tube which is conducting allows quite a large current to flow through its load resistance. This produces a large voltage drop across its load resistance and leaves only a small voltage across the 6SH7 as is indicated in the waveform e_p .

Now consider a signal applied to the grid of the 6SH7 as given by the waveform e_g in Figure 5(b). The introduction of this signal does not change the grid voltages of the switch tubes, e_{g1} and e_{g2} . In this particular case the signal is in phase with e_{g1} and it is seen that the waveform of its plate voltage, e_{p1} , changes very little. Its minimum value is still about 30 volts, the plate voltage of the 6SH7 being so small that even though the signal on its grid is positive it does not cause the current to increase. During the next half-cycle the reference signal cuts off the current allowing e_{p1} to become equal to B+. So the average value of e_{p1} is unchanged.

However, e_{p2} is changed considerably. When the reference signal has its current cut off the plate voltage e_{p2} is 180 volts as before. When the reference signal is drawing grid current and trying to make the plate current large, the signal on the 6SH7 is trying to reduce the current. The end result is that the current is not as large as it was with no signal and the minimum value of e_{p2} is not as small as before. In fact, the 6SH7 causes the current to reduce to such an extent that there is a hump in e_{p2} which occurs when e_g reaches its most negative value. This is shown in the waveform of e_{p2}. The average value of e_{p2} is increased and now there is a difference in the average potentials of the plates which would be indicated by a d.c. meter.

Noise voltages introduced at the grid of the 6SH7 should be of a random nature and should be in phase with e_{g1} just as much as with e_{g2} and therefore should produce the same change in the plate voltages e_{p1} and e_{p2}. The difference in their average values should then be zero. It is the random nature of the noise that allows this detector to differentiate between noise and signal.

If a signal whose frequency differs slightly from the reference voltage frequency is introduced it will be in phase with e_{g1} then with e_{g2} causing the average e_{p2} to increase then the average e_{p1} to increase. The output of this stage as taken between the plates of the switch tubes would have the frequency of the beat note. For low beat frequencies the d.c. meter would follow this beat frequency. In order to filter out these beat frequencies the plate voltages are filtered before being applied to the d.c. meter. This filter consists of sections of resistors and capacitors.

The d.c. meter used here is a vacuum tube meter employing a twin triode and an Esterline-Angus recording milliameter. Using this arrangement, which draws no current from the filter, the filter can be constructed using large resistances. It amounts to filtering each plate voltage, e_{p1} and e_{p2}, then applying them to the grids of a twin triode which has the recording meter connected between its plates.

In adapting the phase-sensitive amplifier for use in the detection of pure nuclear quadrupole resonance, it is necessary to produce a quadrupole resonance signal with the same frequency and phase as the reference signal. Otherwise no signal would be detected through the rather narrow band amplifier.

The fact that a constant magnetic field has the effect of removing the quadrupole resonances has already been mentioned in the discussion of the r-f detection systems. Now by devising a switching method to turn the quadrupole resonance line off and on at the same frequency as the reference signal, it is possible to pass the detected resonance signal thru the phase-sensitive amplifier. These seemingly stringent requirements are satisfied in a not-too-difficult fashion.

Using the modified Livingston detector discussed in Chapter II, frequency modulation with the vibrating condensor was replaced by magnetic modulation using Helmholtz coils to surround the sample. When the oscillator is tuned to resonance this circuit detects absorption modulated at the same frequency as the reference signal. When the oscillator is off resonance the circuit detects no signal. Slow and uniform tuning was provided by driving the variable condenser plates with a small electric motor.

The Helmholtz coils surrounding the sample produce a magnetic field which can be controlled by the type of current passing through the coils. If a pure sine wave current as shown in Figure 6(a) flows through the coils the magnetic field produced by it is zero for only two instants each cycle, i.e., when $i=o$. This means that the time during which resonance is occuring is very short compared to the time during which resonance is not occuring, so that a quadrupole resonance line would not be easily detected in this type of field.

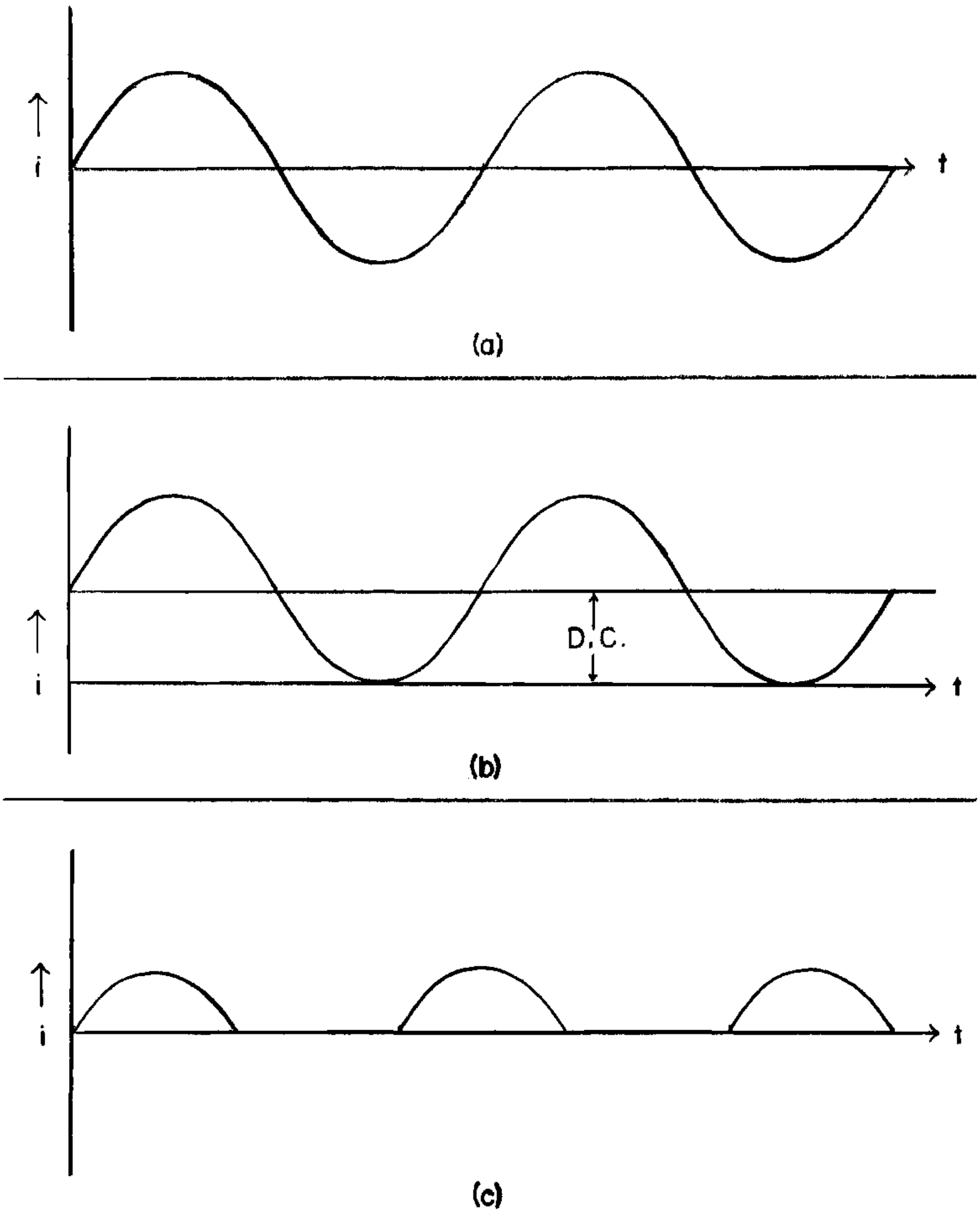


FIGURE 6 – WAVEFORMS FOR HELMHOLTZ COILS

An improvement over this type of field may be made by adding enough d.c. to the sine wave current to give the type of waveform shown in Figure 6(b). Here the magnetic field does not reverse its direction and remains zero for a longer time than in the previous case, giving a higher ratio of resonance time to non-resonance time. Using this type of magnetic field with a preliminary model amplifier, the quadrupole resonance of bromine was detected in sodium bromate with a large signal to noise ratio.

Best results were obtained, however, by using a half-wave rectified sine current having the form given in Figure 6(c). Here the magnetic field is zero for half a cycle meaning that quadrupole resonance is occurring for at least half the time.

Since the magnetic field controls the strength of the quadrupole resonance signal, the field effectively produces a resonance signal having the same frequency and phase as the reference signal.

CHAPTER IV

THE PHASE-SHIFTER

It is conceivable that in travelling the path from the oscillator to the phase-sensitive amplifier some shift in the phase of the signal may take place. Consequently it was desirable to add a device which would shift the phase of the reference signal in order to compensate for any signal phase shift. There are many circuit combinations of resistive, inductive, and capacitive elements which would produce this desired shift, but the circuit shown in Figure 7 was chosen because of its simplicity and because a shift in phase does not appreciably affect the amplitude of the reference signal. This serves to make the shifting of phase an independent variable so that data might be taken relating amplitude of signal to change in phase.

In vector notation, $2\underline{E} = \underline{E}_L + \underline{E}_R$ where $\underline{E}$ is the voltage between the transformer center tap and either side of the transformer coil, $\underline{E}_L$ is the voltage drop across the inductance, and $\underline{E}_R$ is the drop across the resistance. $\underline{E}_L$ and $\underline{E}_R$ are 90° apart in phase so that the vector diagram is of the form shown in Figure 8(a).

The voltages $\underline{E}_L$ and $\underline{E}_R$ are always perpendicular to each other even though their amplitudes may change. The output voltage $\underline{E}'$ of the phase shifter will be constant in magnitude

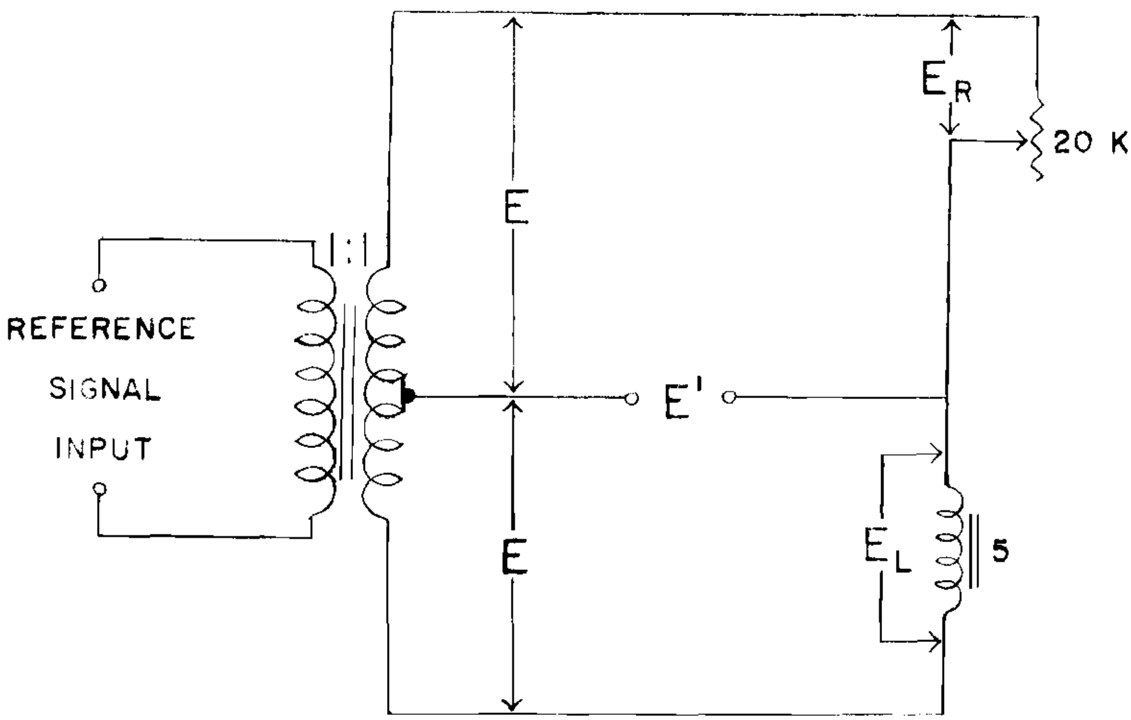


FIGURE 7 - PHASE - SHIFTER

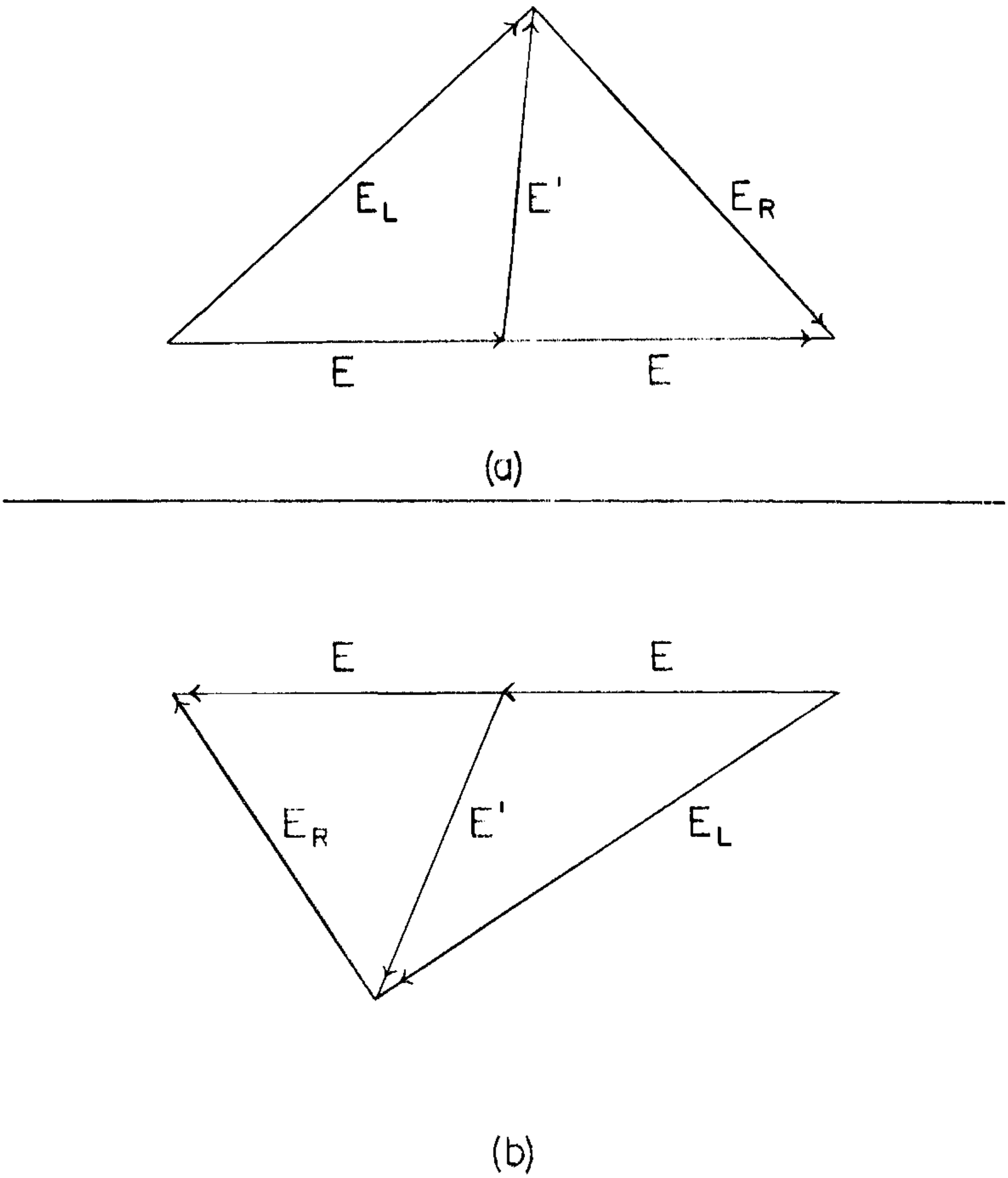


FIGURE 8 - VECTOR DIAGRAM OF PHASE-SHIFTER VOLTAGES

since it is the radius of the circle for which $2\underline{E}$ is the diameter. Notice that when R is zero (so that $\underline{E}_R = 0$) $\underline{E}'$ is in phase with $\underline{E}$. Since the maximum resistance of R is very large compared to the impedance of the inductance, $\underline{E}_R$ can be made very large compared to $\underline{E}_L$. This causes $\underline{E}'$ to be almost 180^0 out of phase with $\underline{E}$. This shift in phase of $\underline{E}'$ with respect to the input signal, initially in phase with $\underline{E}$, may be carried beyond 180^0 by reversing the direction of $2\underline{E}$, as may be accomplished by reversing the wall plug of the Variac transformer which supplies $2\underline{E}$. This puts $\underline{E}_L$ and $\underline{E}_R$ in the bottom half of the circle, Figure 8(b), enabling $\underline{E}'$ to rotate from 180^0 to almost 360^0.

CHAPTER V

SELECTIVE AMPLIFICATION

In addition to providing a shift in phase of the reference signal, it was desirable to provide a stage of selective amplification which would discriminate between signals having the same frequency as the reference signal and signals having other frequencies. This stage is used as part of the preamplification for the phase-sensitive detector, and provides a means of increasing the sensitivity of the overall amplifier.

There are many types of bridged networks in which the combinations of resistive, inductive, and capacitive components used are more sensitive to signals of a given frequency than to signals of other frequencies. The bridged T, the Wein, and the parallel T networks (8) are examples of systems which can be tuned to reject a specified frequency while passing all others. Networks of this type are called "null" networks and in proper conjunction with amplifying stages can provide means of selectively amplifying the frequencies which the networks themselves have been adjusted to reject. The particular null circuit chosen was the "twin T" shown in Figure 9. It is used in an amplifier in such a manner that the signals it passes provide negative feedback. Thus

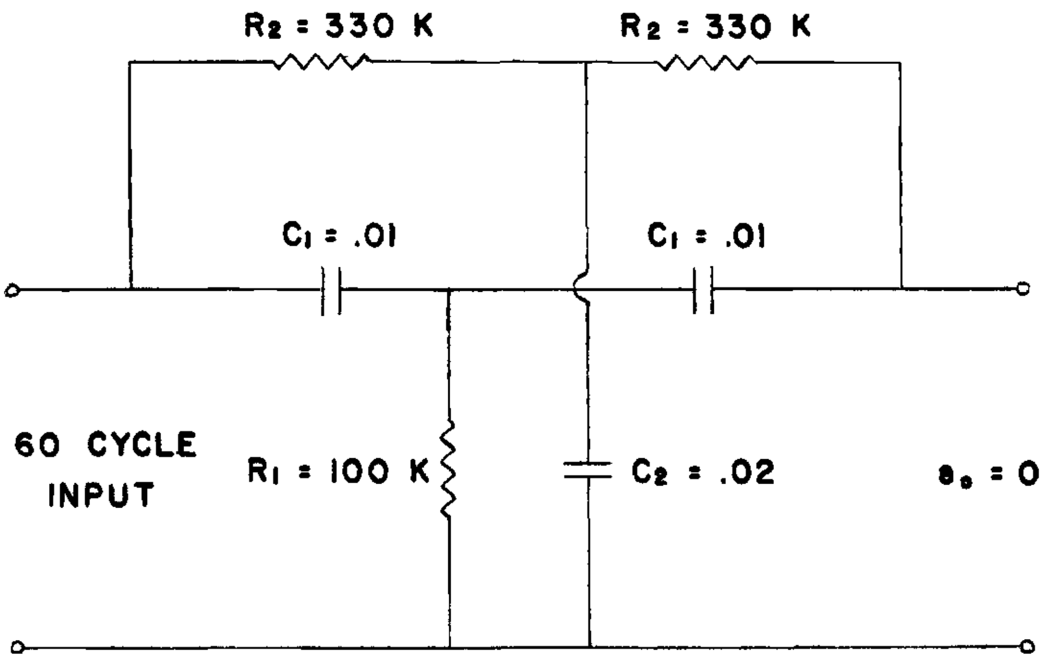


FIGURE 9 – "TWIN-T" NETWORK

maximum amplification of the amplifier occurs at the null frequency.

By requiring that the output voltage, e., equal zero, it can be shown by equating the real and imaginary terms of the transfer impedances that

$$\frac{2}{\omega C_1} = C_2 R_2^2 \omega \tag{12}$$

and

$$\frac{1}{R_1 C_1^2 \omega^2} = 2R_1. \tag{13}$$

Dividing (12) by (13) gives

$$\frac{C_2}{2C_1} = 2\frac{R_1}{R_2} = Q_1^2 . \tag{14}$$

This gives three impedances in terms of the fourth

$$\frac{1}{\omega C_1} = QR_2 , \tag{15}$$

$$2R_1 = Q^2 R_2 , \text{ and} \tag{16}$$

$$\frac{2}{\omega C_2} = \frac{R_2}{Q} . \tag{17}$$

If this constant, Q, is made unity, a network results for which the reactance of the series condensers of the first component T is equal to the resistance of the series arms of the second T component. This requires that the shunt impedances of the two networks are likewise equal, and are equal to half the series impedances. From (16)

$$R_2 = 2R_1 \; , \tag{18}$$

and from (15) and (17)

$$C_2 = 2C_1 \; . \tag{19}$$

The particular values chosen for R_1, R_2, C_1, and C_2 are labeled on Figure 9.

Figure 10 shows how the twin-T network was used on conjunction with a 6SN7 tube which contains 2 triodes in a single envelope. A signal e_1, appearing at grid g_1, is amplified and appears at the upper tube as essentially Ae_1, the total amplification being of the order of A^2 if the two sections are the same. However, the amplification of a signal appearing at g_2 is affected by the cathode "degeneration" of the upper section which is determined by the signal e_1 on grid g_1. This is a desirable effect because the incoming signal can be fed into g_2 while g_1 is used as the high impedance load for the network. The choice of components given in Figure 9, provides a null transmission at 60 cycles per second. Consequently a 60 cycle signal entering into g_2 will not be transmitted through the network back into g_1. This means that there is no degeneration at this frequency and the upper triode, then acts as a conventional triode amplifier whenever a 60 cycle per second signal appears at g_2.

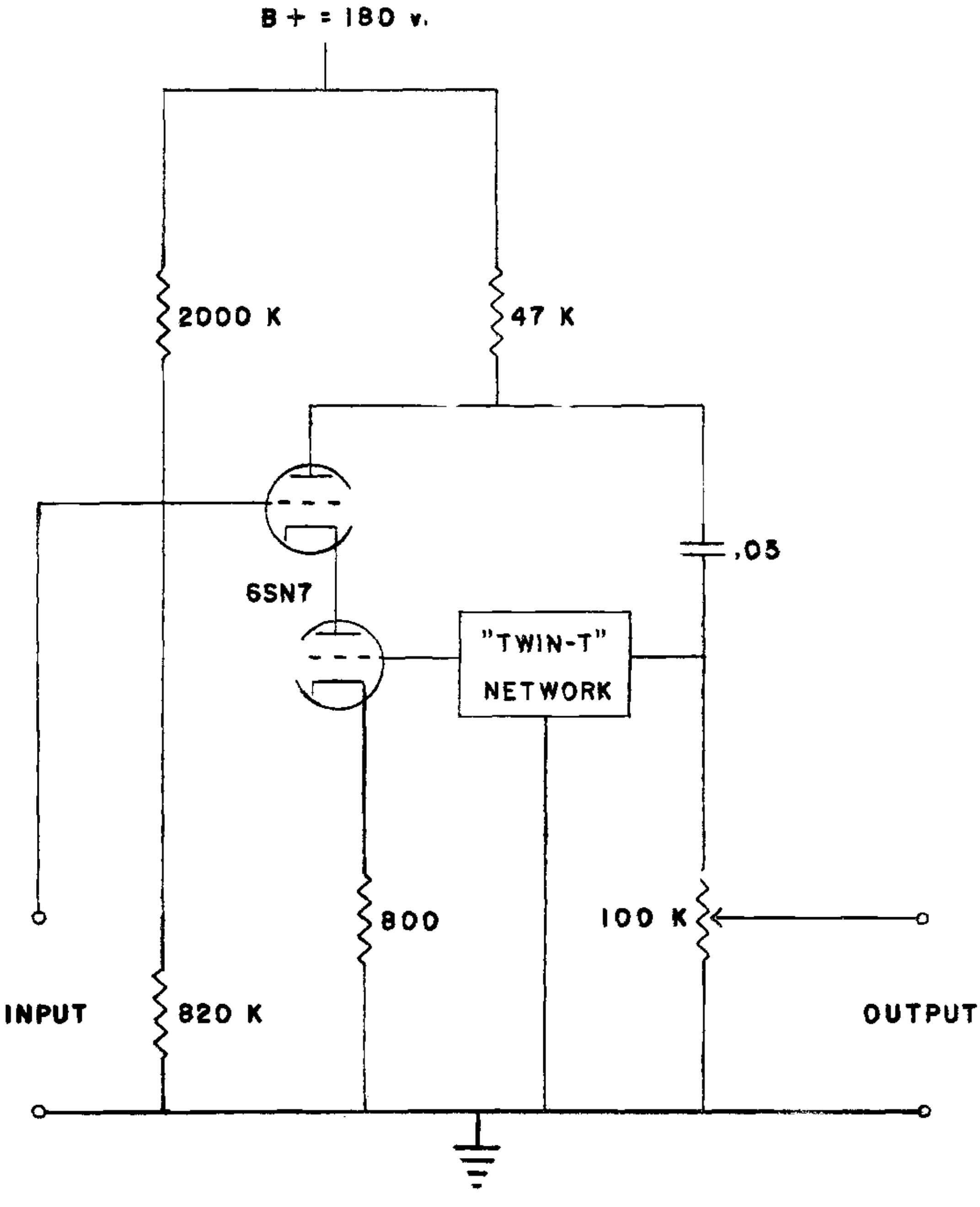


FIGURE 10 – SELECTIVE AMPLIFIER

Consider the positive half cycle of a signal of frequency other than 60 cycles impressed on g_2. This signal is amplified in the upper section and part of the plate voltage drop that is produced is applied to g_1 by the twin-T network. This results in g_1 becoming more negative which serves to increase the cathode to ground potential of the upper triode. This process of degeneration markedly diminishes the amplification of signals of frequencies other than 60 cycles.

In the final arrangement, the signal coming from the simple regenerative detector is amplified once before it enters the tube used in conjunction with the twin-T, and once after leaving the twin-T stage before entering the first tube in the phase-sensitive detector.

CHAPTER VI

THE PHASE - SENSITIVE AMPLIFIER

The selective amplification network, the phase-shifting apparatus, the phase-sensitive detector and the plate and filament voltage source are all housed in the same 11" x 9" x 2" steel chassis. Choices of components used are indicated in the complete diagram shown in Figure 11 as well as in the separate networks shown in Figures 4, 7, 9, and 10. Naturally it was desirable to plan a convenient location for each circuit component so that wiring would be easily facilitated with the restriction that the power supply unit should be as far away from the input signal as possible.

The simple regenerative detector used in conjunction with the phase-sensitive amplifier was a modification of Ralph Livingston's detector described in Chapter I and shown in Figure 2. The modified circuit, including changes in values of circuit components and in method of controlling oscillations, is shown in Figure 12. Provisions were made for cooling the sample by immersion in liquid air and for motor driving the variable condenser plates which served to change the oscillation frequency slowly.

Looking down on the apparatus one sees nine separate tubes, a power supply transformer, two inductances, and two filter condensers. From the front the observer sees three Amphenol sockets, four controls, and a snap switch.

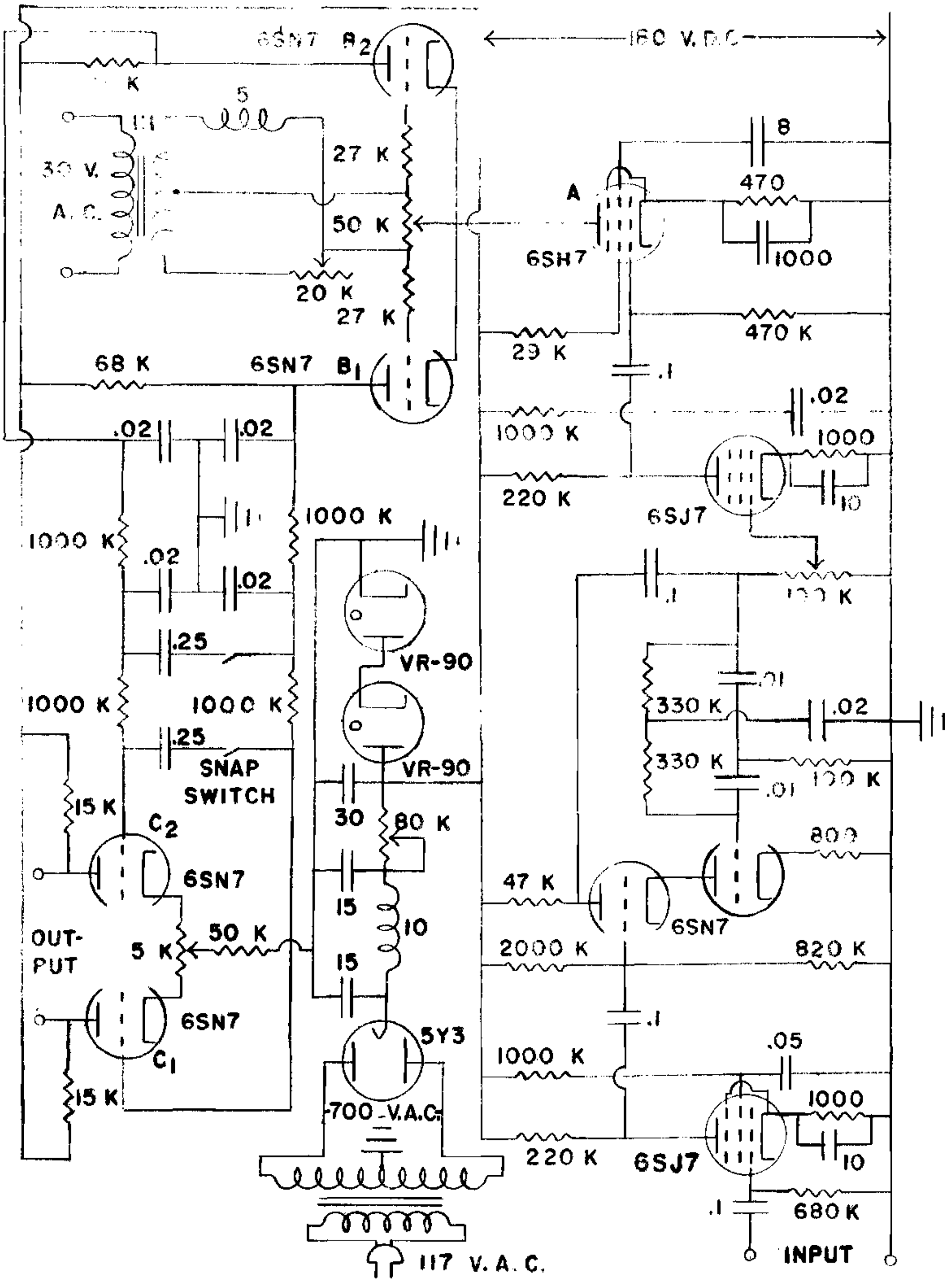


FIGURE 11 – PHASE-SENSITIVE AMPLIFIER

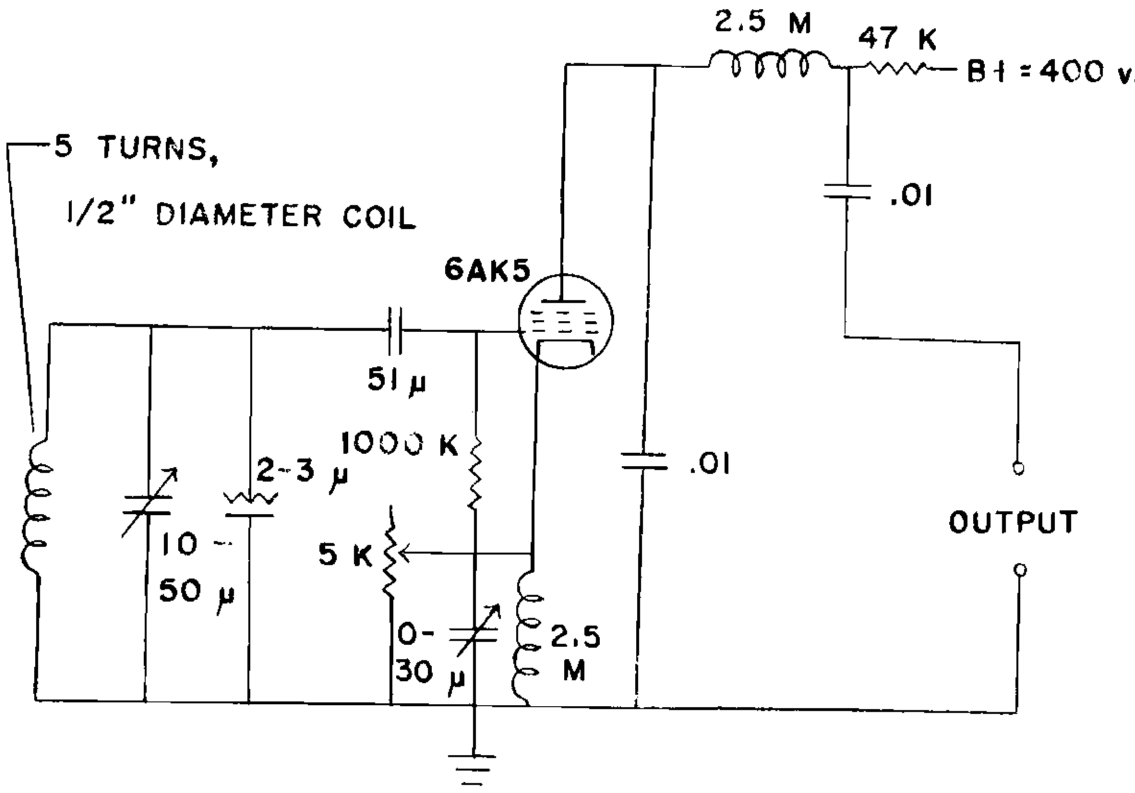


FIGURE 12-MODIFIED LIVINGSTON CIRCUIT

Figure 13 shows a schematic top view of the chassis with preamplification tubes numbered in order of the appearance of the signal, and with phase-sensitive tubes labeled A, B, and C as in Figures 3 and 4. The signal leaves the simple regenerative oscillator and enters tube 1 which is a 6SJ7 pentode amplifier with a voltage gain of approximately 100. The amplified signal enters tube 2, the 6SN7 where frequencies other than 60 cycles are cathode degenerated by the twin-T network, and proceeds into tube 3, another 6SJ7 with a voltage gain of about sixty. Its preamplification is now complete. It next enters the first stage of the phase-sensitive detector, the 6SH7 pentode amplifier, tube A in Figure 4. The output of the phase-detector, which appears between the plates of tube B, is filtered and applied to the grids of C. An Esterline-Angus milliameter recorder is connected between the plates of tube C. The difference in potential between these plates is determined by the difference in the grid potentials of C. Of the three tubes unaccounted for, two are voltage regulating gas tubes, type VR-90, and the third is a high voltage full-wave rectifier type 5Y3.

Viewing the front of the chassis from left to right, the observer sees first an Amphenol input jack into which the signal is fed directly from the simple regenerative detector. Next in order he sees the gain control followed by another Amphenol jack into which the reference signal is fed from a Variac transformer. Following this is the phase-shift control

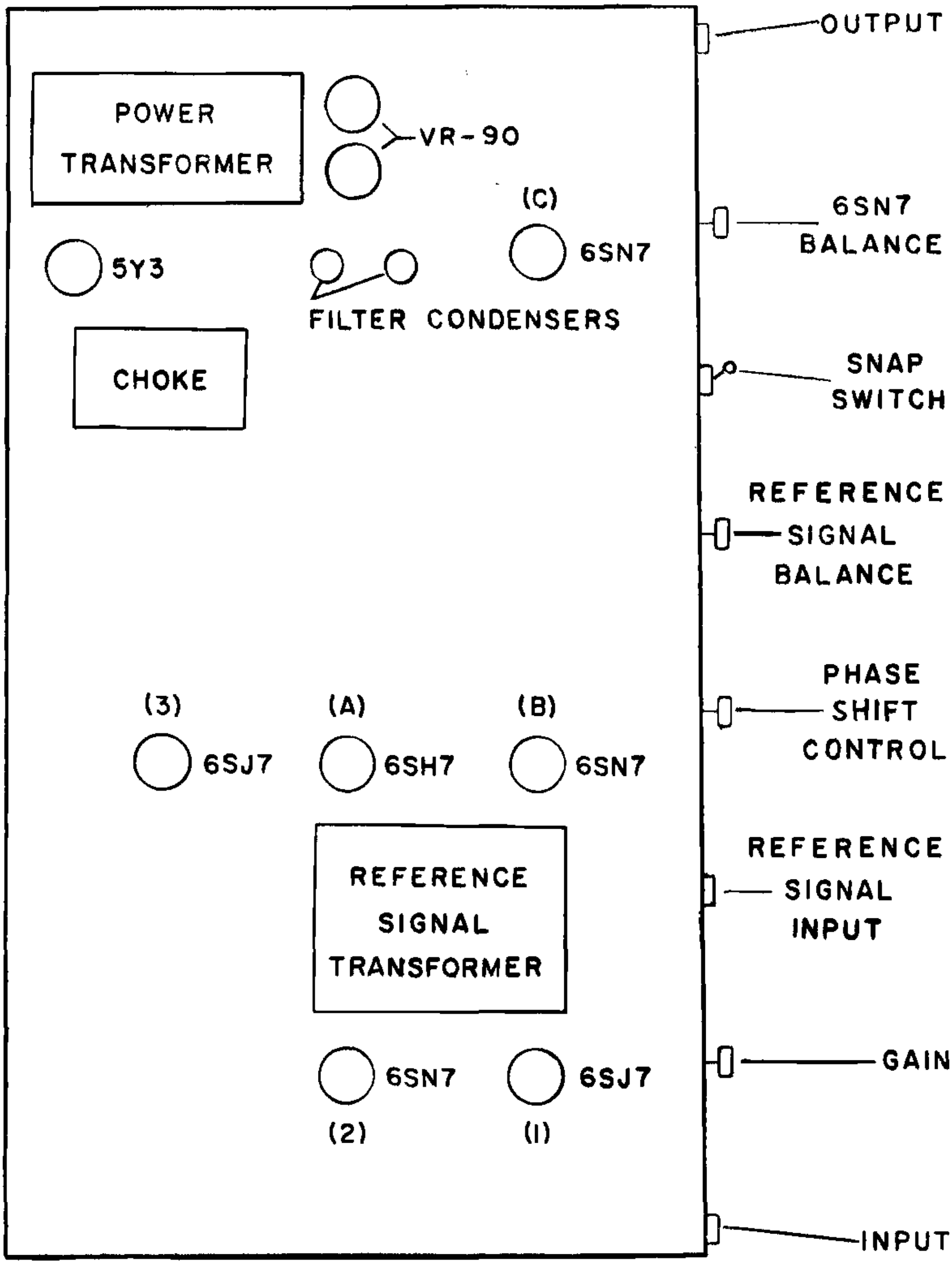


FIGURE 13 - SCHEMATIC TOP VIEW OF CHASSIS

and then the reference signal balance. Next is the snap switch filter control which determines whether the response of the Esterline-Angus recorder shall be fast or slow. The last control is the 6SN7(tube C) balance and the last Amphenol socket contains the output from the plates of C to the Esterline-Angus recorder.

In attempting to test the overall phase-sensitive amplifier, it is best to first observe a known quadrupole resonance line on an oscillograph as described in Chapter I. This preliminary oscillograph viewing assures that the oscillator is functioning properly. A convenient absorption sample is $NaClO_3$ which exhibits a Cl^{35} resonance of about 29.9 megacycles per second at room temperature. Once the line has been detected, the output of the oscillator is introduced into the input of the phase-sensitive amplifier, with the amplifier turned off and with the oscillator moved off the resonant frequency.

After making all connections, the amplifier is turned on with the reference signal turned off. The Esterline-Angus recorder needle is balanced to its original zero position by adjusting the 6SN7 balance control. To obtain the final balance, the reference signal is turned on and the recorder, if now off its zero position, is balanced by adjustment of the reference signal balance control. Any change in the zero position which might result from the changing of other controls should be compensated for by readjusting the 6SN7 balance control without disturbing the reference signal control.

By motor driving the condenser plates of the Livingston type detector in the direction of the resonance line, the Esterline-Angus recorder needle traces out the dip indicating resonance onto the graph paper as the oscillator frequency passes through resonance. It will be observed that the signal-to-noise ratio which appears on the paper is much higher than that viewed previously on the oscillograph.

CONCLUSIONS AND RECOMMENDATIONS

The recording of the multiple lines of Cl^{35} in the compound C_6Cl_6 at liquid air temperatures is shown in Figure 14. These lines were originally found by the author using Livingston's circuit with oscilloscope viewing and have been reported in the Journal of Chemical Physics (9). No comparative oscilloscope photograph is shown since the three lines did not appear on the scope face simultaneously. The signal-to-noise ratio is much larger for each line when the phase-sensitive amplifier is used.

Figure 15(a) is a photograph of the quadrupole resonance of Cl^{37} at liquid air temperature in the compound $NaClO_3$ as viewed on an oscilloscope. Figure 15(b) shows that same resonance line as drawn by the Esterline-Angus recorder when connected to the output of the phase sensitive amplifier. The signal, barely observable on the oscilloscope, is clearly seen on the Esterline-Angus tracing.

Time has permitted testing of the phase-sensitive amplifier at only one reference signal frequency. It is likely that the signal-to-noise ratio might be increased if some other frequency, say 200 cycles, were used to eliminate 60 cycle pickup.

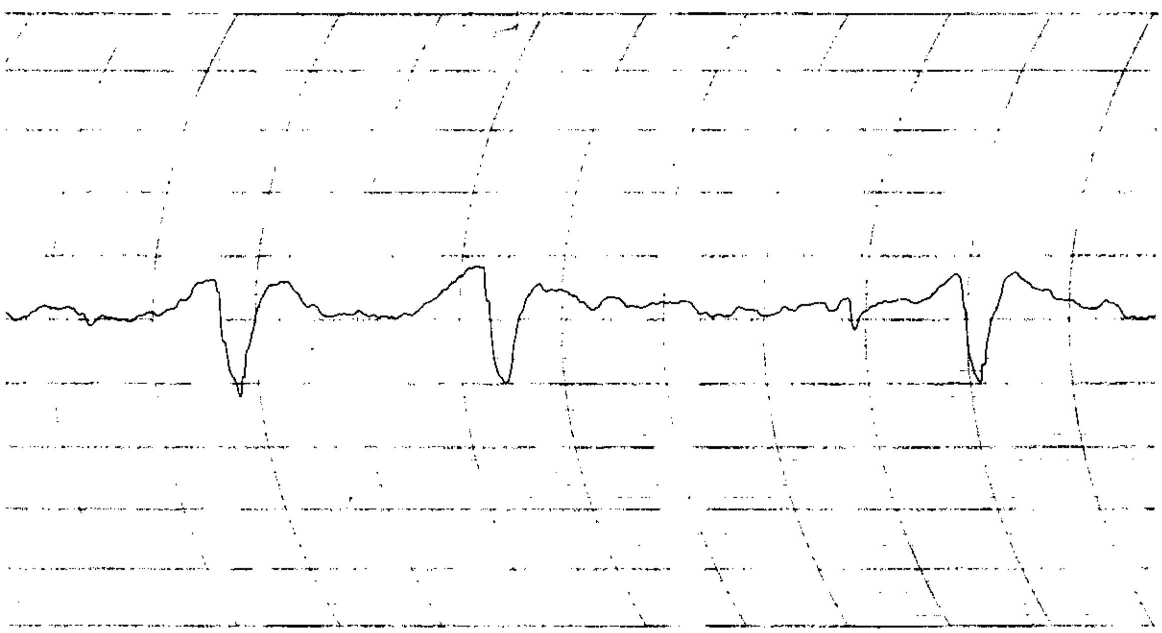
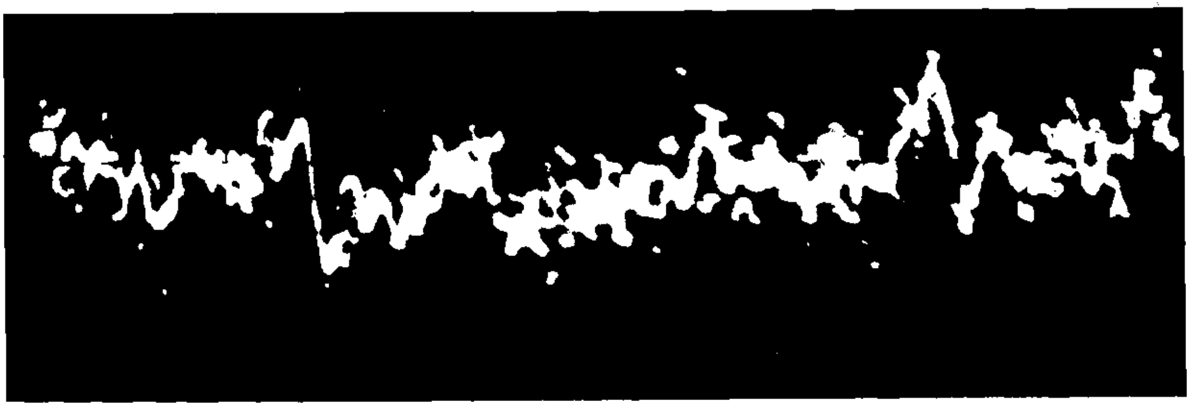
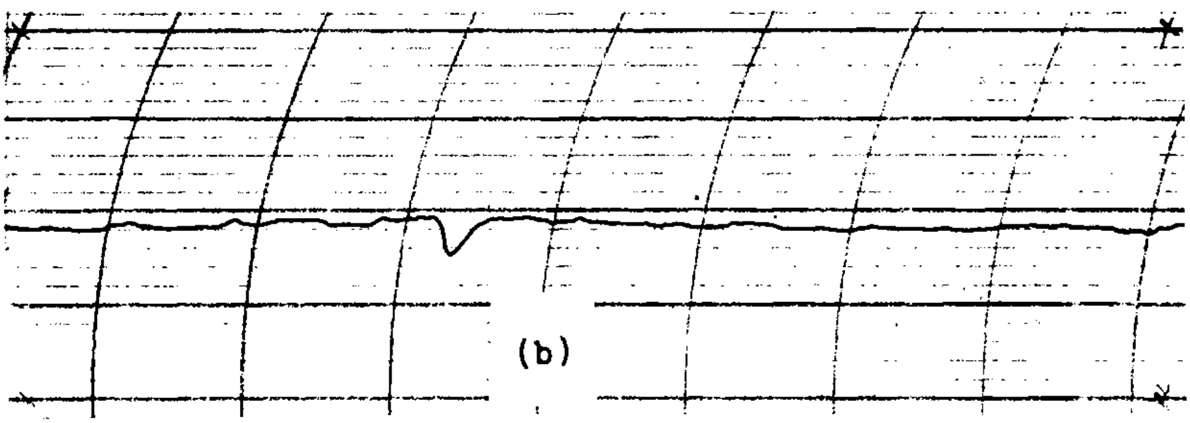


FIGURE 14 - C_6Cl_6 LINES AS SEEN ON RECORDER



(a)



(b)

FIGURE 15 - OSCILLOSCOPE AND ESTERLINE-ANGUS RECORDINGS

APPENDIX A

THE NUCLEAR QUADRUPOLE MOMENT

In Figure 16 the point O is the center of charge of an arbitrary charge distribution. The potential at point P is given by

$$V_P = \int_i \frac{de_i}{R} = \int_i \frac{de_i}{\sqrt{r_i^2 + r_j^2 - 2 r_i r_j \cos\theta_{ij}}} = \int_i \frac{de_i}{r_j\sqrt{1 - \left[2\frac{r_i}{r_j}\cos\theta_{ij} - \left(\frac{r_i}{r_j}\right)^2\right]}} \qquad (20)$$

Writing the denominator of the above as a binomial expansion in terms of the expression in square brackets and collecting equal powers of $\frac{1}{r_j}$ we have

$$V_P = \int_i \frac{de_i}{r_j} + \int_i \frac{r_i \cos\theta_{ij}}{r_j^2}\, de_i + \int_i \frac{r_i^2 (3\cos^2\theta_{ij} - 1)\, de_i}{2 r_j^3} \qquad (21)$$

The first term in this expression

$$V_0 = \frac{1}{r_j} \int_i de_i \qquad (22)$$

is the potential at P produced by the total charge located at O, or the potential due to the electric mono-pole. The second term

$$V_1 = \frac{1}{r_j^2} \int_i r_i \cos\theta_{ij}\, de_i = \frac{1}{r_j^3} \int_i \mathbf{r}_i de_i \cdot \mathbf{r}_j = \frac{\mathbf{m} \cdot \mathbf{r}_j}{r_j^3} \qquad (23)$$

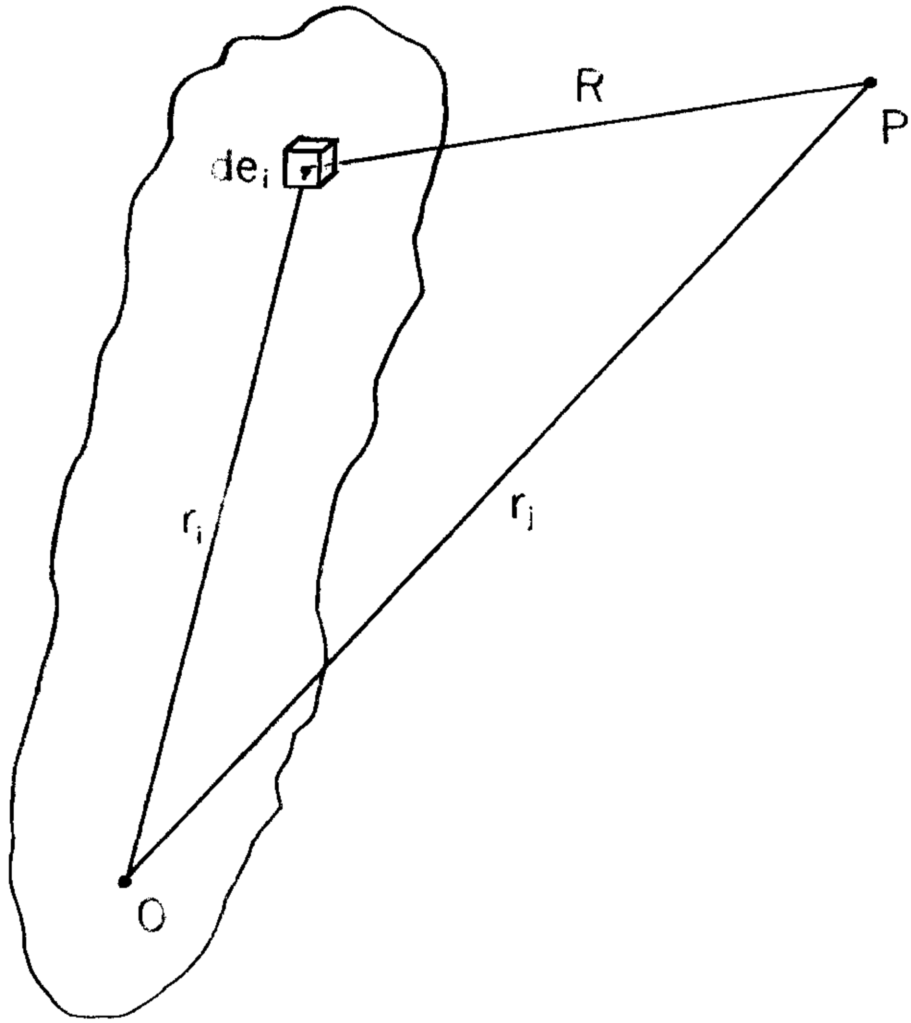


FIGURE 16 – R, P, O, r_i, r_j and de_i RELATIONSHIPS

where $\underline{m} = \int_i \underline{r_i}\, de_i$ (24)

is the potential at P produced by the electric dipole moment of the charge distribution. The third term

$$V_2 = \frac{1}{r_j^3} \int_i \frac{r_i^2}{2} (3\cos^2\theta_{ij} - 1)\, de_i \tag{25}$$

is the potential at P produced by the electric quadrupole moment of the charge distribution.

The charge distribution of a nucleus with spin I is symmetric about the spin axis. For this reason it is convenient to express the integral in equation (25) as a function of θ_{Ii} rather than θ_{ij} (See Figure 17). Making use of the addition theorem for spherical harmonics we write

$$3\cos^2\theta_{ij} - 1 = \tfrac{1}{2}(3\cos^2\theta_{Ij} - 1)(3\cos^2\theta_{Ii} - 1) + \text{terms with } \exp(i\theta_{Ij})\exp(i\theta_{Ii}) \tag{26}$$

when (26) is substituted into (25) and the integration performed one finds the exponential terms average to zero due to the symmetric charge distribution, and we have

$$V_2 = \frac{1}{r_j^3} \int_i \frac{r_i^2}{4} (3\cos^2\theta_{Ij} - 1)(3\cos^2\theta_{Ii} - 1)\, de_i \tag{27}$$

The nuclear quadrupole moment eQ is defined by the integral

$$eQ = \int_i r_i^2 (3\cos^2\theta_{Ii} - 1)\, de_i \tag{28}$$

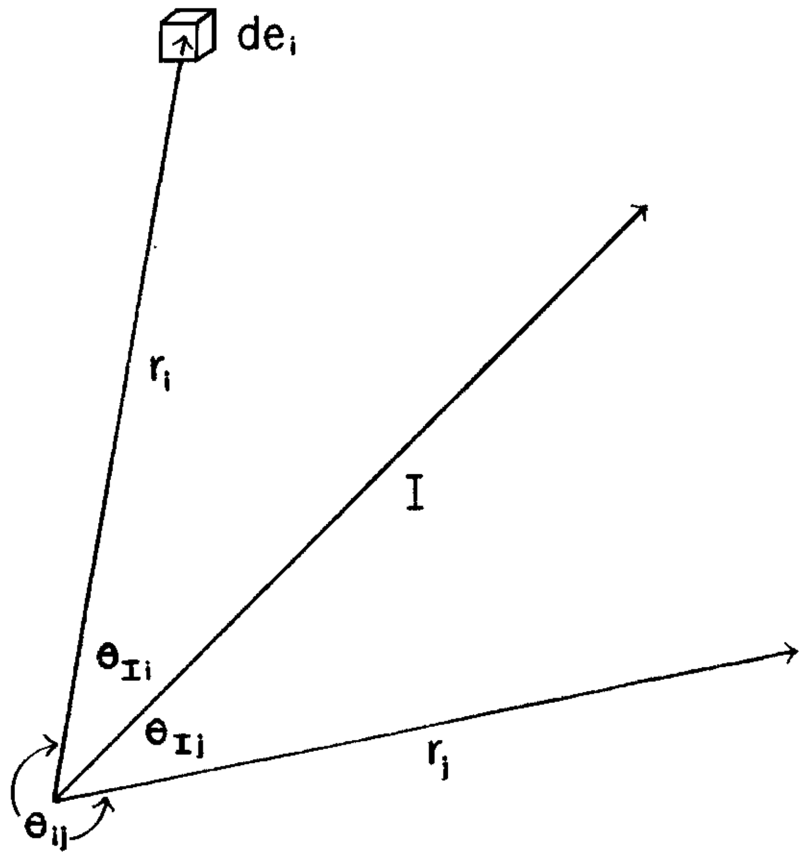


FIGURE 17-I, θ_{ij}, θ_{Ii}, θ_{Ij}, r_i, r_j and de_i RELATIONSHIPS

Thus the potential V_2 may be written

$$V_2 = \frac{eQ(3\cos^2\theta_{Ij} - 1)}{4r_j^3} \tag{29}$$

APPENDIX B

THE NUCLEAR QUADRUPOLE ENERGY

The energy of a nucleus with quadrupole moment eQ in the presence of extranuclear charges e_j may be written

$$E(Q) = \int_j V_2 \, de_j = \frac{eQ}{4} \int_j \frac{(3\cos^2\theta_{Ij} - 1)}{r_j^3} \, de_j \tag{30}$$

If the external charge distribution is symmetric about the z-axis we may with the aid of the addition theorem of spherical harmonics write equation (30) in the form

$$E(Q) = \frac{eQ}{8} \int_j \frac{(3\cos^2\theta_{Iz} - 1)(3\cos^2\theta_{jz} - 1)}{r_j^3} \, de_j \tag{31}$$

Now if we let

$$eQ = \int_j \frac{(3\cos^2\theta_{jz} - 1)}{r_j^3} \, de_j \tag{32}$$

then

$$E(Q) = \frac{e^2 qQ}{8} (3\cos^2\theta_{Iz} - 1) \tag{33}$$

The quantity eq is related to the potential V at the nucleus due to the extranuclear charges as follows:

$$V = \int_j \frac{de_j}{r_j} = \int_j \frac{de_j}{\sqrt{r^2 + z^2}} \tag{34}$$

where the symbols are indentified in Figure 18. By taking successive derivatives of V we obtain

$$\frac{\partial V}{\partial z} = -\int \frac{z\, de_j}{(\sqrt{r^2+z^2})^3} \tag{35}$$

$$\frac{\partial^2 V}{\partial z^2} = \int \left[\frac{3z^2}{(\sqrt{r^2+z^2})^5} - \frac{1}{(\sqrt{r^2+z^2})^3}\right] de_j \tag{36}$$

or

$$\frac{\partial^2 V}{\partial z^2} = \int \frac{(3\cos^2\theta_{jz} - 1)}{r_j^{\,3}}\, de_j \tag{37}$$

From (32) and (37) we see that

$$eQ = \frac{\partial^2 V}{\partial z^2} = V_{zz} \tag{38}$$

and we may write

$$E(Q) = \frac{eq}{8} V_{zz} (3\cos^2\theta_{Iz} - 1) \tag{39}$$

Equation (39) is the classical expression for the quadrupole energy of a charge distribution symmetric about the I axis in an electric field represented by a potential function V symmetric about the z-axis.

According to the quantum theory the allowed values of θ_{Iz} are given by

$$\cos\theta_{Iz} = \frac{m_I}{\sqrt{I(I+1)}} \tag{40}$$

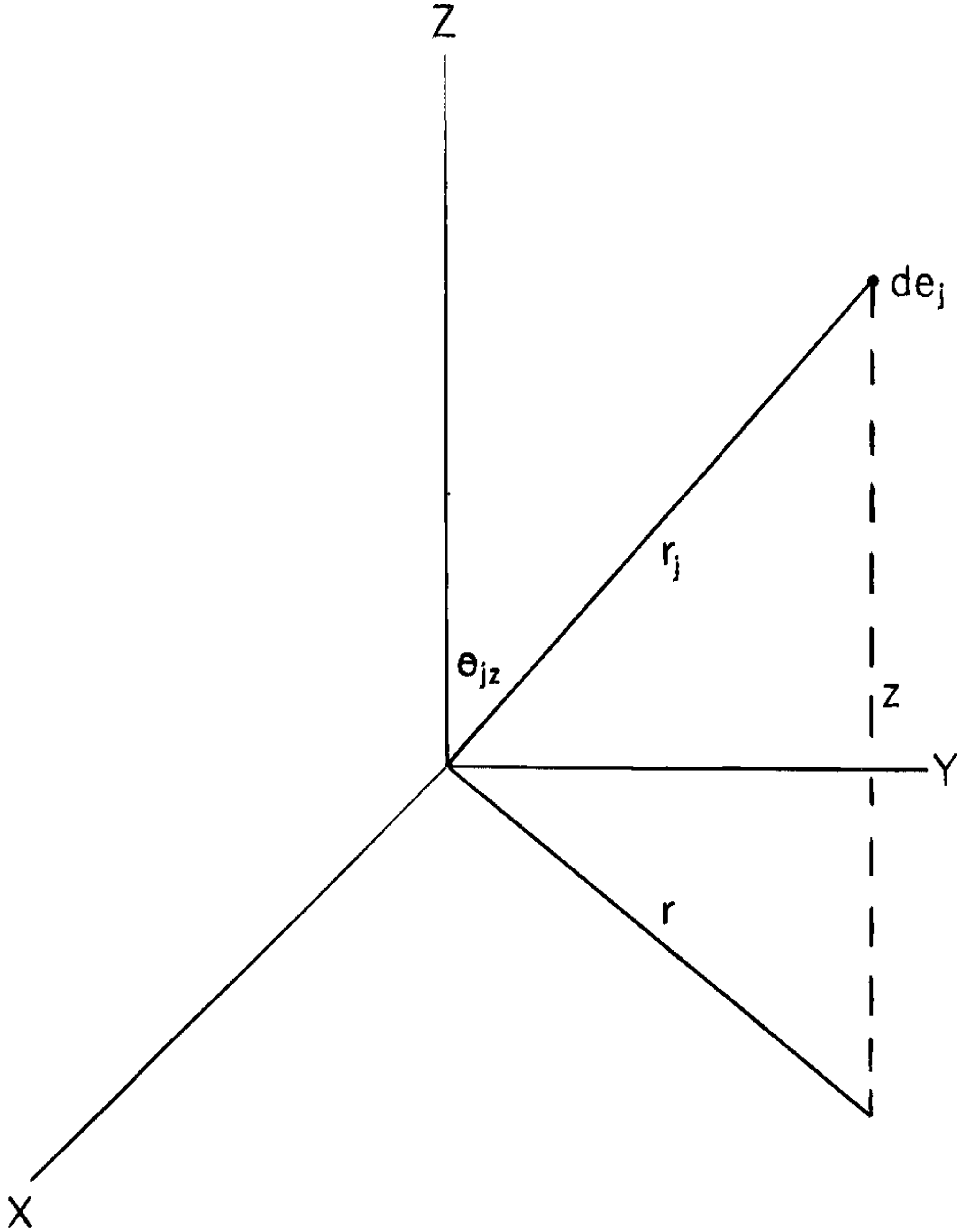


FIGURE 18 - X, Y, Z, θ_{jz}, r, r_j and de_j RELATIONSHIPS

The quantum mechanical expression for the energy may be obtained by substituting (40) into (39) and multiplying by a normalization factor N which will be evaluated by applying the correspondence principle. We then have

$$E(Q) = \frac{eQN}{8} V_{zz} \left[\frac{3 m_I^2}{I(I+1)} - 1 \right] \tag{41}$$

or

$$E(Q) = \frac{eQN}{8} V_{zz} \frac{[3 m_I^2 - I(I+1)]}{I(I+1)} \tag{42}$$

For large quantum number I the quantum mechanical equation (42) should give the same result as the classical equation (39). For large I and $m_I = I$ angle θ_{Iz} is nearly zero. When equation (42) is set equal to (39) and the substitution $\theta_{Iz} = 0$ and $m_I = I$ is made we find that

$$N = \frac{2I(I+1)}{I(2I-1)} \tag{43}$$

With this value of N equation (42) becomes

$$E(Q) = \frac{e(Q)V_{zz}}{4} \frac{3 m_I^2 - I(I-1)}{I(2I-1)} \tag{44}$$

By substituting the allowed values of m_I into equation (44) one obtains the allowed values of the quadrupole energy.

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